MOEMS®
What Every Young Mathlete Should Know 2020-2021

I. VOCABULARY AND LANGUAGE

The following explains, defines, or lists some of the words that may be used in Olympiad problems. To be accepted, an answer must be consistent with both this document and the wording of the problem.

1. **BASIC TERMS**

Sum, difference, product, quotient, remainder, ratio, square of a number (also, perfect square), factors of a number. The ***value*** of a number is the simplest name for that number. "**Or**" is inclusive: "*a* or *b*" means "*a* or *b* or both."

Note: ***Difference*** between two values is the distance between them and can be computed by subtracting the smaller value from the larger. E.g. The "difference between 4 and 6" is 2 as is "the difference between 6 and 4".

DIVISION M: Square root of a number, cube of a number (also, perfect cube). The relation between power and root.

2. **READING**

Read "1 + 2 + 3 + ..." as "one plus two plus three and so forth" (without end).
Read "1 + 2 + 3 + ... + 10" as "one plus two plus three and so forth *up to ten.*"

3. **STANDARD FORM OF A NUMBER**

a. The standard form of a number refers to the form in which we usually write numbers (also called Hindu-Arabic numerals or positional notation).

b. A digit is any one of the ten numerals 0,1,2,3,4,5,6,7,8,9. Combinations of digits are assigned place values in order to write all numbers. A number may be described by the number of digits it contains: 358 is a three-digit number. The "lead-digit" (leftmost digit) of a number is not counted as a digit if it is 0: 0358 is a three-digit number. Terminal zeros of a number are the zeros to the right of the last nonzero digit: 30,500 has two terminal zeros because to the right of the digit 5 there are two zeros. The number zero is a one-digit number.

c. Increasing (or decreasing) order of digits of a number: If the digits of a number are in "increasing" order, when reading the number from left to right (the only acceptable way of reading a "number"), each digit with the higher place value is less than (and NOT equal to) any other digit of a lesser place value (e.g, 12345 is a 5-digit number whose digits are in "increasing" order; 1223 is NOT in increasing order.

d. The "digit sum" of a whole number is the total of its individual digits; thus the digit sum of 123 is 6.

Example and note:

Example: "How many 3-digit numbers" have a digit sum of 4? Answer: 10.
The 10 are: 103, 112, 121, 130, 202, 211, 220, 301, 310, and 400.
Note: The hundreds' digit cannot be zero.

4. **SETS AND SETS OF NUMBERS**

a. **"Set" vs "Ordered List"** A set is an unordered collection of members. The sets {1,2,3}, {1,3,2}, {2,1,3}, {2,3,1}, {3,1,2}, and {3,2,1} are all ways of writing the same set. Whereas the ordered lists 1,2,3 and 1,3,2 are different.

*Note: An **Ordered List** is also called a **Sequence.***

b. **Set quantifiers "all", "none", and "some".** The quantifier **"some"** indicates that one or more element satisfies the condition stated. **"Some"** is the opposite of "none" and could be stated as "at least one…"

Examples: Considering the "ONE DIGIT WHOLE NUMBERS LESS THAN 3"= {0,1,2},
*only {0,1,2} is "**ALL** ONE DIGIT WHOLE NUMBERS LESS THAN 3",*
*only { } is "**NONE** OF THE ONE DIGIT WHOLE NUMBERS LESS THAN 3",*
*but all of {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, and {0,1,2} are considered "**SOME** OF THE ONE DIGIT WHOLE NUMBERS LESS THAN 3"*

c. **Counting Numbers** = {1, 2, 3, ...}.
d. **Whole Numbers** = {0, 1, 2, 3, ...}
e. **Integers** = {... , –3, –2, –1, 0, +1, +2, +3, ...}.
f. The terms **positive numbers**, **negative numbers**, **nonnegative numbers**, **nonpositive numbers**, and **nonzero numbers**.
g. **Consecutive Numbers** are counting numbers that differ by 1, such as 83, 84, 85, 86, and 87.
h. **Consecutive Even Numbers** are multiples of 2 that differ by 2, such as 36, 38, 40, and 42.
i. **Consecutive Odd Numbers** are nonmultiples of 2 that differ by 2, such as 57, 59, 61, and 63.
j. **Palindrome**, **palindromic number**, or **numeral palindrome** is a whole number that remains the same when its digits are reversed. Like 16461 and 6336, for example, it is "symmetrical". The smallest palindrome is 0. Single digits (e.g. 5) and repeated digits (e.g. 777) are all palindromes.

5. MULTIPLES, DIVISIBILITY, AND FACTORS

a. The product of two whole numbers is called a **multiple** of each of them. Zero is considered a multiple of every whole number. *Example: Multiples of 6 = {0,6,12,18,24,30,…}.*
Note: many but not all authorities expand the definition of multiple to include all integers. To them –24 is a multiple of 6. ***Olympiad problems will define multiples as counting numbers.***
b. A whole number ***a*** is said to be **divisible by** a counting number ***b*** if *b* divides *a* with zero remainder. In such instances: (1) $\frac{a}{b}$ is equal to a whole number, (2) *b* is called a **factor** of *a*, and (3) *a* is called a multiple of *b*.

6. NUMBER THEORY

a. A **prime number** (also, a prime) is a counting number that has exactly two different factors, namely the number itself and the number 1. *Examples: 2, 3, 5, 7, 11, 13,….*
b. A **composite number** is a counting number which has at least three different factors, namely the number itself, the number 1, and at least one other factor. *Examples: 4, 6, 8, 9, 10, 12,….*
c. The number 1 is neither prime nor composite since it has exactly one factor, namely the number itself. Thus, there are three separate categories of counting numbers: prime, composite, and the number 1.
d. A number is **factored completely** when it is expressed as a product of prime numbers.
Example: 144 = 2×2×2×2×3×3. It may also be written as 144 = $2^4 \times 3^2$.
e. The **Greatest Common Factor (GCF)** of two counting numbers is the largest counting number that divides each of the two given numbers with zero remainder.
Example: GCF(12,18) = 6.
f. If the GCF of two numbers is 1, then we say the numbers are **relatively prime** or **co-prime**.
g. The **Least Common Multiple (LCM)** of two counting numbers is the smallest counting number that each of the given numbers divides with zero remainder.
Example: LCM(12,18) = 36.

7. FRACTIONS

a. A **common fraction** is a fraction in the form $\frac{a}{b}$ where *a* is a whole number and *b* is a counting number. One meaning is *a* ÷ *b*.

DIVISION M: **Common fractions** include negative values as well as zero and positive. In this case for the common fraction $\frac{a}{b}$, *a* is an integer and *b* is a nonzero integer.

Note that $-\left(\frac{a}{b}\right) = \frac{-a}{b} = \frac{a}{-b}$ *although only the first two forms could be considered as being in simplest form.*

b. A **unit fraction** is a common fraction with numerator 1.

c. A **proper fraction** is a common fraction in which a < b. Its value is between 0 and 1.

d. An **improper fraction** is a common fraction in which a ≥ b. Its value is 1 or greater than 1. A fraction whose denominator is 1 is equivalent to an integer.

e. A **complex fraction** is a fraction whose numerator or denominator contains a fraction.

Examples: $\dfrac{\frac{2}{3}}{5}, \dfrac{7}{\frac{3}{4}}, \dfrac{\frac{5}{4}}{\frac{5}{8}}, \dfrac{3-\frac{1}{3}}{3+\frac{1}{3}}$

f. The fraction is **simplified** if a and b have no common factor other than 1 [**GCF(*a*,*b*) = 1**]. **DIVISION M:** As noted in 7.a. above, the denominator of a fraction in simplest form cannot be negative.

g. A **decimal** or **decimal fraction** is a fraction whose denominator is a power of ten. The decimal is written using decimal point notation.

Examples: $\frac{7}{10}$ *= .7; .36, .005, 1.4*

h. **DIVISION M:** A **percent** or **percent fraction** is a fraction whose denominator is 100, which is represented by the percent sign.

Examples: $\frac{45}{100}$ *= 45%; 8%, 125%, 0.3%*

8. STATISTICS AND PROBABILITY

a. The **average (arithmetic mean)** of a set of N numbers is the sum of all N numbers divided by N.

b. The **mode** of a set of numbers is the number listed most often.

c. The **median** of an **ordered set** of numbers is the middle number if N is odd, or the mean of the two middle numbers if N is even.

d. The **probability** of an event is a value between 0 and 1 inclusive that expresses how likely an event is to occur. It is often found by dividing the number of times an event *does* occur by the total number of times the event *can* possibly occur.

Example: The probability of rolling an odd number on a standard die is $\frac{3}{6}$ *or* $\frac{1}{2}$*. Either fraction is acceptable as a correct probability.*

9. GEOMETRY

a. **Angle**: degree-measure, vertex, congruent; acute, right, obtuse, straight, and reflex angles.

b. **Congruent segments** are two line segments of equal length.

c. **Polygons**, **circles**, and **solids**:

Parts: side, angle, vertex, diagonal; interior region, exterior region.

Triangles: acute, right, obtuse; scalene, isosceles, equilateral.

*Note: an equilateral triangle is isosceles with **all** three sides congruent.*

Quadrilaterals: diagonal; parallelogram, rectangle, square, trapezoid, rhombus.

*Note(1): a square is **one type of rectangle** with all four sides congruent. It is also a rhombus with all four angles congruent.*

*Note(2): a parallelogram is **one type of trapezoid** with both pairs of opposite sides parallel.*

Others: cube, rectangular solid; pentagon, hexagon, octagon, decagon, dodecagon, icosagon.

Perimeter: the number of unit lengths in the boundary of a plane figure.

Area: the number of unit squares contained in the interior of a region.

Circumference: the perimeter of a circular region.

Congruent figures: two or more plane figures all of whose corresponding sides are congruent and all of whose corresponding angles are congruent.

Similar figures: two or more plane figures whose size may be different but whose shape is the same.

Note: all squares are similar; all circles are similar.

Volume: the number of unit cubes contained in the interior of a solid.

Surface Area: the sum of the areas of all the faces of a geometric solid.

DIVISION M: Geometric Solid, Right Circular Cylinder, face, edge.

II. SKILLS

1. COMPUTATION

a. The tools of arithmetic are needed for problem solving. Competency in the basic operations on whole numbers, fractions, and decimals is essential for success in problem solving at all levels. In **DIVISION M** competency in basic operations on integers and signed numbers should be developed.

b. **Order of Operations**. When computing the value of expressions involving two or more operations, the following priorities must be observed from left to right:

1) Do operations in parentheses, braces, or brackets first, working from the inside out,
2) Do multiplication and division from left to right, and then
3) Do addition and subtraction from left to right.

Example: $3 + 4 \times 5 - 8 \div (9 - 7)$

$= 3 + 4 \times 5 - 8 \div 2$

$= 3 + 20 - 4$

$= 19$

2. ANSWERS

Unless otherwise specified in a problem, equivalent numbers or expressions should be accepted. For example, $3\frac{1}{2}$, $\frac{7}{2}$, and 3.5 are equivalent.

Units of measure generally are not required in answers but must be correct if given in an answer. More generally, an answer in which any part is incorrect is not acceptable. Students should be careful to include *only* required information to prevent credit from being denied. While an answer that differs from the official one can be appealed, credit can be granted only if the wording of the problem allows for an alternate interpretation or if it is flawed so that no answer satisfies all conditions of the problem.

Measures of area are usually written as square units, sq. units, or units^2. For example, square centimeters may be abbreviated as sq cm, or cm × cm, or cm^2. In **DIVISION M**, cubic measures are treated in a like manner.

After reading a problem, a wise procedure is to indicate the nature of the answer at the bottom of a worksheet before starting the work necessary for solution.

Examples: "*A* = ___, *B* = ___"; "*The largest number is* __".

Another worthwhile device in practice sessions is to require the student to write the answer in a simple declarative sentence using the wording of the question itself.

Example: "*The average speed is 54 miles per hour*."

This device usually causes the student to reread the problem.

3. MEASUREMENT

The student should be familiar with units of measurement for time, length, area, and weight (and for **DIVISION M**, volume) in English and metric systems. Within a system of measurement, the student should be able to convert from one unit to another.

III. SOME USEFUL THEOREMS

1. If a number is divisible by 2^n, then the number formed by the last n digits of the given number is also divisible by 2^n; and conversely.
   Example: 7,292,536 is divisible by 2 (or 2^1) because 6 is divisible by 2.
   Example: 7,292,536 is divisible by 4 (or 2^2) because 36 is divisible by 4.
   Example: 7,292,536 is divisible by 8 (or 2^3) because 536 is divisible by 8.

2. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3.
   If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.
   Example: 323,745 is divisible by 3 because 3+2+3+7+4+5 = 24 which is a multiple of 3.
   Example: 658,773 is divisible by 9 because 6+5+8+7+7+3 = 36 which is a multiple of 9.

3. A number is divisible by 5 if its units' digit is 5 or 0.

4. A number is divisible by 11 if the difference between the sum of the odd-place digits and the sum of the even-place digits is 0 or a multiple of 11.
   Example 90,728 is divisible by 11 because (9+7+8) – (0+2) = 24 – 2 = **22**, which is a multiple of 11.

5. If A and B are natural numbers, then:
   (i) GCF(A,B) × LCM(A,B) = A × B.
   (ii) LCM(A,B) = (A × B) ÷ GCF(A,B).
   (iii) GCF(A,B) = (A × B) ÷ LCM(A,B).
   Example: If A = 9 and B = 12: GCF(9,12) = 3, LCM(9,12) = 36, A × B = 9 × 12 =108.
   Then: (i) 3 x 36 = 108; (ii) 108 ÷ 3 = 36; (iii) 108 ÷ 36 = 3.

6. If p represents a prime number, then p^n has $n + 1$ factors. Example: 2×2×2×2×2 = 2^5 has 6 factors which are 1, 2, 2×2, 2×2×2, 2×2×2×2, 2×2×2×2×2. In **exponential form**, the factors are: 1, 2, 2^2, 2^3, 2^4, and 2^5. In **standard form**, the factors are: 1, 2, 4, 8, 16, and 32. Notice that the factors of 2^5 include both 1 and 2^5.

   Problem: *how many factors does 72 have?* 72 = 2×2×2×3×3 = $2^3×3^2$. Since 2^3 has 4 factors and 3^2 has 3 factors, 72 has 4×3 = 12 factors. The factors may be obtained by multiplying any one of the factors of 2^3 by any one of the factors of 3^2: (1, 2, 2^2, 2^3) × (1, 3, 3^2). Written in order, the 12 factors are: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

IV. SOME GENERAL STRATEGIES FOR PROBLEM SOLVING

Draw a picture or diagram	*Make an organized list*
Solve a simpler problem	*Work backward*
Find a pattern	*Make a table*
Guess, check and revise	*Use reasoning (logic)*

Encourage students to *guess, check, and revise* when no other method presents itself. With time and practice, more efficient strategies should start to present themselves.

Thorough discussions of these and many other useful topics may be found in all volumes of *Math Olympiad Contest Problems* and in *Creative Problem Solving in School Mathematics*.