Division **E**

Mathematical Olympiads

November 13, 2018

for Elementary & Middle Schools

Contest **1**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: ____

1A Find the sum: 2 + 4 + 6 + 8 + 10 + 20 + 40 + 60 + 80 + 100.

1B The "digit sum" of a whole number is the total of its individual digits; thus, the digit sum of "123" is 6. How many 3-digit numbers have a digit sum of 4? [Note: The hundreds digit cannot be 0.]

1C Chloe has a rectangle made of construction paper. She folds the rectangle in half to form another rectangle. She folds the resulting rectangle in half to form a 3-cm-by-3-cm square. What is the number of square centimeters in the area of Chloe's original rectangle?

Name: ____

Answer Column
1A
1B
1C
1D
1E
Do Not Write in this Space. *For PICO's Use Only.* *SCORE:*

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1D In the multiplication AB × BA = 57B, A and B represent different digits, AB and BA are 2-digit numbers and 57B is a 3-digit number.
If AB < BA, what is the 2-digit number AB?

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1E Find the least multiple of 41 whose only digits are 1.

Division **E**

Mathematical Olympiads

December 11, 2018

for Elementary & Middle Schools

Contest **2**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: ____

2A What is the value of 7643 + 6437 + 4376 + 3764?

2B Hannah is thinking of a five-digit number. The number reads the same forward as backward. The product of Hannah's number and 101 is 2488842. What is Hannah's number?

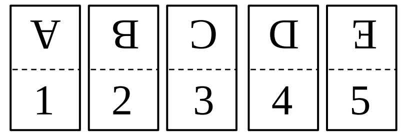
2C A group of people planned to go on a trip using 12 buses. The number of people on each bus was the same. They realized they didn't need quite so many buses, so they cancelled 2 of the buses and redistributed all the people equally among the remaining buses. Each bus ended up with 5 additional people. How many people were taking the trip?

Name: ____

Answer Column
2A
2B
2C
2D
2E
Do Not Write in this Space. For PICO's Use Only. *SCORE:*

– Page may be folded along dotted line. –

2D The place cards shown are folded along the dotted line so that only a number or letter is visible. Chrissy enters the room and sees all five place cards, some with numbers showing and some with letters showing. The numbers that she sees add up to 10. How many different sets of numbers are possible?



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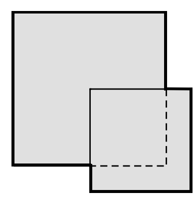
2E In the figure, the octagon is formed by overlapping two squares. The overlapping region is also a square. The larger square has a side length of 6 cm. The smaller square has one corner at the center of the larger square, and the smaller square has a side length of 4 cm. How many centimeters is the perimeter of the octagon?



Division **E**

Mathematical Olympiads

January 15, 2019

for Elementary & Middle Schools

Contest **3**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: ____

3A Compute the simplified value of
$3 \times 5 + 5 \times 7 + 7 \times 9 + 9 \times 11$.

3B Ava's PIN code is a 4-digit number. The sum of the four digits is 22. Reading from left to right, the first and second digits are the same. The second digit is twice the third digit. The first digit is four times the fourth digit. What is Ava's PIN code?

3C When two people play a game, each player starts with 10 points. The winner of each round gets 3 points and the loser of each round loses 3 points. William and Abigail play the game. William wins exactly four rounds, and Abigail ends up with 16 points. How many rounds did they play altogether?

Name: ______________________________

Answer Column
3A
3B
3C
3D
3E
Do Not Write in this Space. For PICO's Use Only. *SCORE:*

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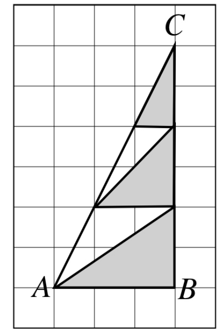
– Page may be folded along dotted line. –

3D What fraction of the right triangle, ΔCAB, is shaded? The right angle is angle B and each little box in the grid is a unit square.



3E In the cryptarithm shown, different letters represent different digits. If two letters are the same, they represent the same digit. What is the greatest value that GOOSE could be?

$$\begin{array}{r} \text{D U C K} \\ \underline{+\,\text{D U C K}} \\ \text{G O O S E} \end{array}$$

Division **E**

Mathematical Olympiads

February 12, 2019

for Elementary & Middle Schools

Contest **4**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: ____

4A What is the value of 30 – 8 + 40 – 11 – 22 – 19?

4B Emma is filling in the 3 by 3 grid shown with counting numbers from 1 through 9. She writes them, one per box, making sure the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers in each diagonal are the same. The grid shows the numbers she has placed so far. Which number must go in the box labeled X?

	9	X
	5	7
	1	

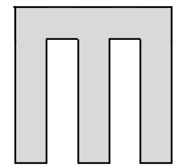
4C The five-digit number A6A6A is divisible by 11. What is the digit A?

Name: ______________________________

Answer Column
4A
4B
4C
4D
4E
Do Not Write in this Space. For PICO's Use Only. *SCORE:*

– Page may be folded along dotted line. –

4D A MOEMS-tile is shaped like an M as shown. It is a 5 by 5 square with two 4 by 1 rectangles removed. Jimmy is playing a game where the object is to place as many MOEMS-tiles as possible on a 6 by 30 game board without any overlap. What is the maximum number of tiles Jimmy can place on this board?



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4E In the cryptarithm shown, different letters represent different digits. If two letters are the same they represent the same digit. When A = 5 and O = 4, what is the greatest value that WIN could be?

$$\begin{array}{r} T\ I\ C \\ T\ A\ C \\ +\,T\ O\ E \\ \hline W\ I\ N \end{array}$$

Division **E**

Mathematical Olympiads

March 5, 2019

for Elementary & Middle Schools

Contest **5**

Directions to Students: After all questions have been read by your PICO, you will have 30 minutes to complete this contest. You may not have a pen or pencil in your hand while the PICO reads the set of questions to the class. Calculators are not permitted. All work is to be done on the pages provided. No additional scrap paper is to be used. Answers must be placed in the corresponding boxes in the answer column.

Name: ______________________________

5A Find the sum: 54321 + 65432 + 76543 + 87654 + 98765.

5B There is a method to square a number that ends with the digit "5".
$25 \times 25 = 625$ because "2" times "2 + 1" = 6 and $5 \times 5 = 25$.
$35 \times 35 = 1225$ because "3" times "3 + 1" = 12 and $5 \times 5 = 25$.
$45 \times 45 = 2025$ because "4" times "4 + 1" = 20 and $5 \times 5 = 25$.
They always end with a "25".
Evaluate 555×555.

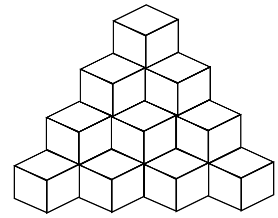
5C The 5-digit number 2A13A is divisible by 99.
What digit is A?

Name: ______________________________

Answer Column
5A
5B
5C
5D
5E
Do Not Write in this Space. For PICO's Use Only. *SCORE:*

– Page may be folded along dotted line. –

5D Isabella has stacked unit cubes in a corner of a room as shown in the picture. If Isabella continues the pattern so that the stack is five cubes high instead of four cubes high, how many more cubes would she need?



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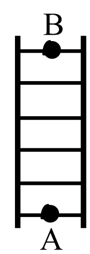
5E A ladder has 6 rungs. An ant is going to climb the ladder, without retracing any part of its path, to get to the top rung of the ladder. Given that the ant starts at A, the middle of the bottom rung, how many different ways can the ant get to point B, the middle of the top rung?



Division **E** | Mathematical Olympiads | November 13, 2018 | for Elementary & Middle Schools | Contest **1**

SOLUTIONS AND ANSWERS

1A	330
1B	10
1C	36
1D	14
1E	11111

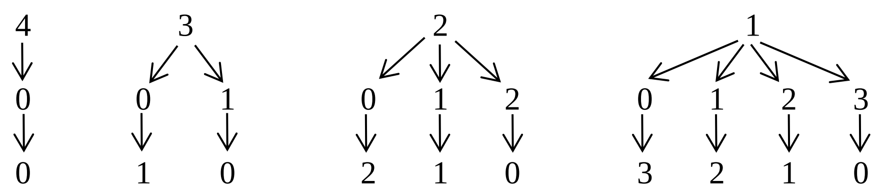
1A **METHOD 1** *<u>Strategy</u>: Group terms to get common sums.*
Notice, the sum of 2 and 8 is 10, and the sum of 4 and 6 is 10, so 2 + 4 + 6 + 8 + 10 = 3(10) = 30. Similarly, 20 + 80 = 40 + 60 = 100, so
20 + 40 + 60 + 80 + 100 = 3(100) = 300. Thus, the total sum is 30 + 300 = **330**.

METHOD 2 *<u>Strategy</u>: Use place value.*
Add 2 + 4 + 6 + 8 + 10 = 30, Therefore 20 + 40 + 60 + 80 + 100 = 300.
300 + 30 = 330.

FOLLOW UP: Find the sum: 100 – 80 + 60 – 40 + 20 – 10 + 8 – 6 + 4 – 2. [54]

1B **METHOD 1** *<u>Strategy</u>: Make an organized list of three-digit numbers with a digit sum of 4.*
Begin with the least possible three-digit number, 100. Since the hundreds digit is 1, the tens and ones digits must sum to 3; this means they must be 0 and 3 or 1 and 2, resulting in 103, 130, 112, or 121. If the hundreds digit is 2, the tens and ones digits must sum to 2; this means they must be 0 and 2 or 1 and 1, resulting in 202, 220, or 211. Similarly for hundreds digits of 3 or 4, the resulting numbers are 301, 310, or 400. There are **10** three-digit numbers that have a sum of 4.

METHOD 2 *<u>Strategy</u>: Make a tree diagram listing all possible 3-digit numbers.*
The hundred's digit can only be 1, 2, 3 or 4.

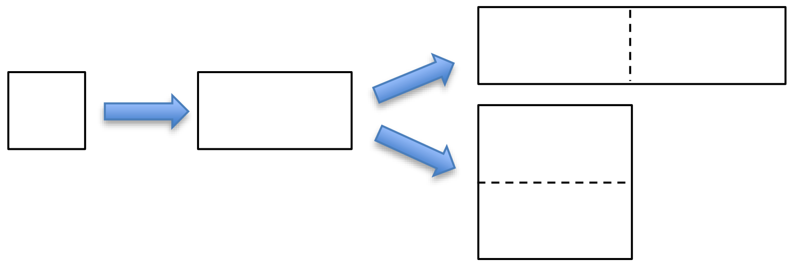


There are 10 possible 3-digit numbers such that the sum of the digits is 4.

FOLLOW UPS: (1) How many three-digit numbers have a digit sum of 5, 6, and 7? [15, 21, 28] *(2) How many four-digit numbers have a digit sum of 4?* [20]

1C **METHOD 1** *<u>Strategy</u>: Work backwards from the final figure.*
The final resulting rectangle is a 3-cm-by-3-cm square; therefore it has an area of 9 cm^2. Since the original rectangle was folded in half, twice, the final rectangle can be "un-folded" twice, to get back to the original. If it is unfolded once, the area will double, meaning 2(9) = 18 cm^2. If it is unfolded once more, the area will double again, that is 2(18) = **36** cm^2.

METHOD 2 *<u>Strategy</u>: Draw pictures.*



The last square, is 3×3, which means the original rectangle was either a square or a rectangle. The dimensions of the original rectangle were either 6×6 or 3×12. In either case the area is 36 cm^2.

FOLLOW UP: What is the greatest possible perimeter that the original rectangle could have? [30 cm]

1D METHOD 1 *<u>Strategy</u>: Use properties of multiplication to make educated guesses.*
Ask, what two 2-digit numbers when multiplied get you close to 570? We know that $20 \times 30 = 600$, which is too big. Therefore, one of the numbers is less than 20. Start with $15 \times 51 = 765$, which is also too big. Try $14 \times 41 = 574$. The two digits 1 and 4 meet the requirements of the problem. So AB =**14**.

METHOD 2 *<u>Strategy</u>: Consider the multiplication procedure.*

Since the product is a 3-digit number, the value of A × B must be less than 10. Consider the 9 possible values for the digit B in the number 57B. If B = 1, A = B = 1 which is not possible. If B = 2, the numbers are 21 and 12 and the product is too small. If B = 3, the numbers are 31 and 13, still too small. When B = 4, the numbers must be 41 and 14. Since $41 \times 14 = 574$, AB = 14.

	A	B
	× B	A
	A×A	A×B
B×A	B×B	
B×A	(A×A+B×B)	A×B

FOLLOW UP: In the multiplication AB × BA = A45B, A and B represent different digits. If AB < BA what is the two-digit number AB? [18]

1E METHOD 1 *<u>Strategy</u>: List numbers with only ones as the digits and check for divisibility by 41.*
The least possible number whose only digit is the number 1, and is greater than 41, is 111. However, 111 is not divisible by 41 and neither is 1111. When 11,111 is divided by 41 the result is 271, which means **11,111** is the least multiple of 41 whose only digit is the number 1.

METHOD 2 *<u>Strategy</u>: Divide 41 into 11111… until a remainder of '0' is a result.*

Start a division problem using 111 as the initial dividend. If a non-zero remainder results, append another 1 to the dividend. Continue this process until the remainder is zero. The final dividend is the solution.

$$\begin{array}{r} 271 \\ 41\,\overline{)\,11111\ldots} \\ \underline{82} \\ 291 \\ \underline{287} \\ 41 \\ \underline{41} \\ 0 \end{array}$$

FOLLOW UPS: (1) What is the next multiple of 41 whose only digits are 1? [1,111,111,111]. *(2) Find the least multiple of 42 whose only digits are 4.* [444,444]

> ***NOTE:*** *Other FOLLOW-UP problems related to some of the above can be found in our three contest problem books and in "Creative Problem Solving in School Mathematics."*
> *Visit <u>www.moems.org</u> for details and to order.*

Division **E** | Mathematical Olympiads | December 11, 2018 | for Elementary & Middle Schools | Contest **2**

SOLUTIONS AND ANSWERS

2A	**22,220**
2B	**24,642**
2C	**300**
2D	**3**
2E	**28**

2A **METHOD 1** *Strategy: Use patterns found in each place value.*

The ones, tens, hundreds, and thousands place digits each consist of 3, 7, 4, and 6, in some order. So, using 3 + 7 + 4 + 6 = 10 + 10 = 20 in each place value, the problem becomes 20 + 200 + 2000 + 20000, which equals **22,220**.

METHOD 2 *Strategy: Trade off values to simplify the problem.*

In each place value, the digits are 3, 4, 6, and 7. The 7 can give 2 to the 3 to make them both 5s without changing the total value. The 6 can give 1 to the 4 to make them both 5s without changing the total value. If this happens in all 4 place values, the overall value is 5555 × 4 = 22,220.

FOLLOW UPS: (1) What is the value of 3517+5173+1735+7351? [17,776]
(2) What is the value of 93,517+35,179+51,793+17,935+79,351? [277,775]

2B **METHOD 1** *Strategy: Estimate and then make adjustments.*

2488842 divided by 100 is 24888.42 which 1) must be close to Hannah's number and 2) have the first two digits the same as Hannah's number. Thus

Hannah's number is of the form 24?42. We also know that because 8 is in both the hundreds and ten thousands place, and 2 + 6 = 8, the ? = 6. Hannah's number is **24642**.

$$\begin{array}{r} 24?4200 \\ +\ 24?42 \\ \hline 2488842 \end{array}$$

METHOD 2 *Strategy: Do the division.*

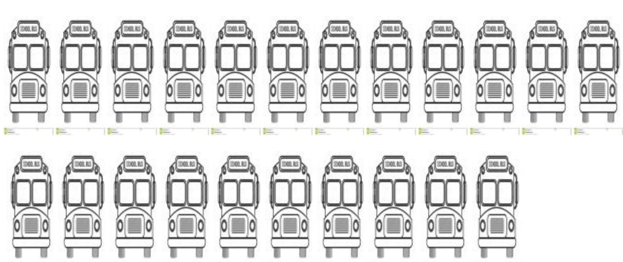
Divide Hannah's number (the product) by the divisor 101 to get the quotient. 2488842 ÷ 101 = 24642.

FOLLOW UP: 101, 24642, and their product are all palindromes. Under what condition will the product of two palindromes not produce a palindrome? [Condition: the product forces a place value regrouping (or "carried" digits).]

2C **METHOD 1** *Strategy: Draw a picture representation (tape diagram).*

12 Buses

10 Buses



+5 +5 +5 +5 +5 +5 +5 +5 +5 +5 = 50 more people

Since 50 people were moved, 25 people were moved from each of the two cancelled buses. Therefore, there were originally 25 people per bus and 25 × 12 = **300** people taking the trip.

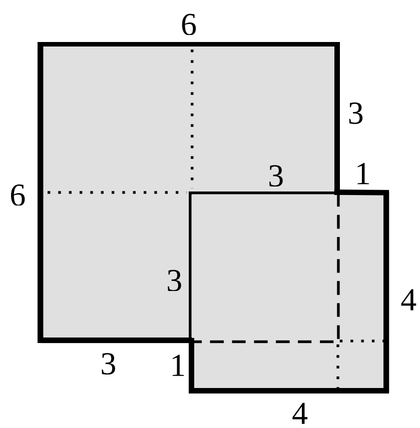
METHOD 2 Strategy: *Create an equation and solve it.*
Let N = the number of people on each of the 12 buses. Then use either of the two equations:
$12N - 50 = 10N$ or $12N = 10N + 50$. In each case $N = 25$ and $12N = 12 \times 25 = 300$.

METHOD 3 Strategy: *Use the properties of common multiples.*
The total number of people can be evenly divided by 12 and by 10. Listing multiples of 10 and 12, we find the common multiples 60, 120, 180, … or more simply multiples of 60. The number of people being moved off of buses is $5 \times 10 = 50$, so the total number must also be a multiple of 50. The common multiples of 50 and 60 are 300, 600, 900, … each of which is a multiple of 300. Checking the numbers we see that 300 is the perfect match. 12 buses × 25 people = 10 buses × (25 + 5) people.

FOLLOW UP: Suppose: 1) The people going to the event were comprised of couples 2) No couple could be split between 2 buses and 3) no single bus could hold more than 65 people. What is the fewest number of buses needed to get the 300 people to the event with the same number of couples on each bus? [5]

2D Strategy: *Simplify the problem, using the largest possibilities first.*
Since 5 is the greatest number and can be used only once, find other numbers that add up to five so that together they will equal 10. 5 + 4 + 1 is one combination and 5 + 3 + 2 is the other. Then, try using 4 as the highest number. Only 4 + 3 + 2 + 1 = 10 works. There are **3** sets of different numbers possible.

2E **METHOD 1** Strategy: *Determine the lengths of each side of the octagon.*



Since the overlap is a square and one of its vertices is at the center of the 6 × 6 square, we see that the length of a side of the overlapping square is 3. We can now use subtraction to determine the length of each side of the octagon.

Add the lengths (in cm) of each side of the octagon.
6 + 6 + 3 + 1 + 4 + 4 + 1 + 3 = **28**.

METHOD 2 Strategy: *Subtract the perimeter of the overlapping square from the sum of the perimeters of the two bigger squares.*
The perimeter of the square whose side is 6 cm is 6 × 4 = 24 cm.
The perimeter of the square whose side is 4 cm is 4 × 4 = 16 cm.
The perimeter of the overlapping square whose side is 3 cm is 3 × 4 = 12 cm.
Therefore the perimeter of the octagon is 24 + 16 – 12 = 28 cm.

FOLLOW UPS: (1) Find, in cm^2, the area of the entire octagon. [43 cm^2] *(2) Find the area of the smallest square possible that can surround the entire octagon.* [49 cm^2]

Division E | Mathematical Olympiads January 15, 2019 for Elementary & Middle Schools | Contest 3

SOLUTIONS AND ANSWERS

3A **METHOD 1** *Strategy: Evaluate using the order of operations.*
$3 \times 5 + 5 \times 7 + 7 \times 9 + 9 \times 11 = 15 + 35 + 63 + 99 =$ **212**.

METHOD 2 *Strategy: Factor out common factors.*

$$\begin{aligned} 3 \times 5 + 5 \times 7 + 7 \times 9 + 9 \times 11 &= 5 \times (3 + 7) + 9 \times (7 + 11) \\ &= 5 \times 10 + 9 \times 18 \\ &= 50 + 162 \\ &= 212 \end{aligned}$$

FOLLOW UP: What is twice the value of $9 + 7 \times 7 + 5 \times 5 + 3 \times 3 + 2$? [188]

3B **METHOD 1** *Strategy: Make a table working backwards.*
Since the first digit is four times the fourth digit, the fourth digit can only be a 1 or a 2. Create a table of values for each case.

4th digit	1st digit	2nd digit	3rd digit	Sum
1	4	4	2	11
2	8	8	4	22

Therefore, Ava's pin is **8842**.

METHOD 2 *Strategy: Solve algebraically.*
Let N represent the fourth number. Then the first and second numbers are each 4N and the third number is 2N. It follows that 4N + 4N + 2N + N = 22. Hence 11N = 22 and N = 2. Ava's pin is 8842.

FOLLOW UP: A locker combination consists of 5 different single digit numbers. The second number is twice the first; the third number is 50% more than the second; the fourth number is 1 more than the first; and the fifth number is 2 more than the fourth. Find the combination. [24635]

3C **METHOD 1** *Strategy: Consider the number of wins and losses for Abigail.*
Since William wins 4 rounds, Abigail must lose 4 rounds. Abigail starts with 10 points and ends with 16 points. She must win 4 rounds to get back to 10 points and then win 2 additional rounds to increase her points by 6 to get a total of 16 points. Since she must win 4 + 2 = 6 rounds and she loses 4 rounds, they must have played 6 + 4 = **10** rounds.

METHOD 2 *Strategy: Create a table to display the different rounds.*
The order of the wins and losses is unimportant.

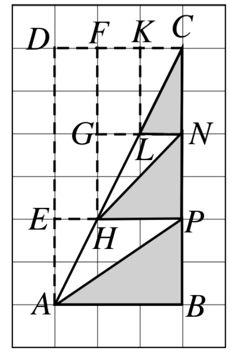
Round	Start	1	2	3	4	5	6	7	8	9	10
Winner	----	W	W	W	A	A	A	W	A	A	A
William's points	10	13	16	19	16	13	10	13	10	7	4
Abigail's points	10	7	4	1	4	7	10	7	10	13	16

They must play 10 rounds.

Answers
3A 212
3B 8842
3C 10
3D 2/3 or equivalent
3E 18872

FOLLOW UP: In a variation of the game, the players still start with 10 points each. The winner of each round gets 5 points while the loser of each round loses only 3 points. William and Abigail play the game. Abigail loses exactly 3 rounds. At the end of the game, William has exactly 10 points. How many points does Abigail have at the end of the game? [26]

3D **METHOD 1** *Strategy: Use the formula for area of a right triangle.*



Find the area of each shaded triangle using A = (1/2) × base × height. The base and height of a right triangle are the legs of the triangle.
The area of ΔCLN = (1/2)(1)(2) = 1.
The area of ΔNHP = (1/2)(2)(2) = 2.
The area of ΔPAB = (1/2)(3(2) = 3.
The sum of the areas of the three shaded triangles is 1 + 2 + 3 = 6.
Since the area of ΔCAB = (1/2)(3)(6) = 9, the fractional part ΔCAB that is shaded is 6/9 = **2/3**.

METHOD 2 *Strategy: The area of a right triangle is ½ the area of a rectangle.*
Draw horizontal and vertical lines to form rectangles as seen in the diagram.
The area of ΔCAB = ½ the area of rectangle *CDAB* = (1/2)(3)(6) = 9.
The area of ΔCLN = ½ the area of rectangle *CKLN* = (1/2)(1)(2) = 1.
The area of ΔNHP = ½ the area of rectangle *NGHP* = (1/2)(2)(2) = 2.
The area of ΔPAB = ½ the area of rectangle *PEAB* = (1/2)(3)(2) = 3.
Therefore, the part of ΔCAB that is shaded is (1 + 2 + 3)/9 = 6/9 = 2/3.

FOLLOW UP: Find the areas of each of the two unshaded triangles inside ΔCAB. [1, 2]

3E *Strategy: Reason using number sense.*
Make a list of the possible numbers to be used and cross them off once used: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Notice that the only possible value for G is 1 so cross off 1 from the list. We want the greatest possible sum so let D = 9 which means that O = 8. This means that U + U = 2U must be less than 10. Since O = 8, U = 4. The list now contains the numbers 0, 2, 3, 5, 6, and 7. To maximize the word GOOSE, we want S = 7, the greatest of the remaining numbers. That means that C = 3. The sum in the units column must be greater than or equal to 10. The only numbers remaining for that to occur are 5 and 6. We want the greatest value for GOOSE so let K = 6 making E = 2. The greatest value for GOOSE is **18872**, which will occur when DUCK is 9436.

FOLLOW UP: In the given cryptarithm, find the least possible value for GOOSE. [16654]

NOTE: *Other FOLLOW-UP problems related to some of the above can be found in our three contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.*

SOLUTIONS AND ANSWERS

4A	10
4B	2
4C	4
4D	10
4E	796

4A **METHOD 1** *<u>Strategy</u>: Reason logically by combining the positive and negative addends separately and then find the sum of the two results.*

<u>Step 1</u> Rearrange the expression: 30 + 40 – 8 – 22 – 19 – 11.
<u>Step 2</u> Combine the positives: 30 + 40 = 70.
<u>Step 3</u> Combine the negatives: (–8) + (–22) + (–19) + (–11) = – 60.
<u>Step 4</u> Add the results: 70 + (–60) = **10**.

METHOD 2 *<u>Strategy</u>: Add and subtract from left to right.*
30 – 8 = 22; 22 + 40 = 62; 62 – 11 = 51; 51 – 22 = 29; and 29 – 19 = 10.

FOLLOW UP: What is the value of 10 + 18 + 32 + 45 – 19 – 31 – 10 – 25 – 20? [0]

4B **METHOD 1** *<u>Strategy</u>: Use deductive reasoning to find the missing numbers.*
The sum of <u>column 2</u> is 15. Therefore, all rows, columns, and diagonals must sum to 15. The numbers in <u>row 1</u> must add to 15. Since one number is 9, the other two numbers must add to 6. The unused numbers are 2, 3, 4, 6, and 8 and the only ones that add to 6 are 2 and 4. If X = 4 then the missing number in <u>column 3</u> would also be 4 which is not possible. Therefore, X = **2**.

4	9	2
3	5	7
8	1	6

METHOD 2 *<u>Strategy</u>: Consider the sums along the diagonals.*
The sum of the numbers in all directions must be 15. The number 5 is central to the puzzle. Therefore, the diagonally opposite corners must contain numbers that will sum to 10. The only possibilities are 9 + 1, 8 + 2, 7 + 3, 6 + 4. However, the 9 and 7 are already used, leaving us only 8 + 2 and 6 + 4. The 8 cannot go in the lower right corner, the upper left corner, or the upper right corner because that would immediately cause a sum to be greater than 15. The 8 must go in the lower left corner, causing the upper right corner to be 2. Therefore, X = 2.

FOLLOW UP: Solve the original problem after interchanging the location of the numbers 9 and 1. [6]

4C **METHOD 1** *<u>Strategy</u>: Use the divisibility test for the number 11.*
A number is divisible by 11 if the difference between the sum of the odd-place digits and the sum of the even-place digits is divisible by 11 or equal to zero. The number A6A6A will be divisible by 11 if A – 6 + A – 6 + A = 3A – 12 is divisible by 11. The only single-digit number that satisfies this is A = **4**.

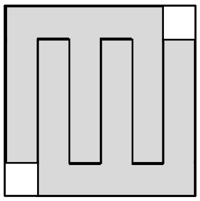
METHOD 2 *<u>Strategy</u>: Make a chart of possible solutions.*

Possible value for A	1	2	3	4
Divisible by 11?	No	No	No	Yes 4224

Therefore A = 4.

FOLLOW UP: What is the value of the digit X in the number 2,458,X64 if it is divisible by 99? [7]

4D **METHOD 1** *Strategy: Combine 2 tiles to make 1 new tile that is 6 by 6.*



Flip one MOEMS-tile upside down and then fit the tile together with a second MOEMS-tile to form a 6 by 6 square tile with two of the corners missing. Since the game board is 6 by 30, we can arrange 5 of these square tiles on the board. Therefore, there will be **10** of the MOEMS-tiles on the board.

METHOD 2 *Strategy: Determine the area of a tile and compare it to the area of the board.*
The game board has an area of 180 square units. The area of a MOEMS-tile is 17 square units. The maximum number of tiles that can be accommodated is 10 (180 ÷ 17 = 10 with a remainder of 10). A duplicated tile rotated 180°, can interlock with the original tile resulting in a 2 square unit loss of coverage for the pair. Therefore, the maximum number of tiles possible is 10.

FOLLOW UP: Determine the perimeter of one of the MOEMS-tiles. [36]

4E *Strategy: Reason using number sense.*
Make a list of the possible numbers to be used and cross them off once use: 0, 1, 2, 3, 6, 7, 8, and 9. Since the goal is to make WIN as great as possible we want W to be 9. This can only happen if T = 3.

Consider the tens column. Since "I" is both an addend and the sum, the tens column must add to a 2-digit number so the regrouping will add 1 to the hundreds column forcing a 4-digit final sum. Therefore, we reject W = 9.

If W = 8, then T = 2 and we need I to make the sum I + 5 + 4 be at least 20. This is not possible so reject W = 8.

If W = 7, then T = 2 and I + 5 + 4 must be greater than 9 so that regrouping adds a 1 to the hundreds column. Since we want WIN to be as great as possible let I = 9. We now have 29C + 25C + 24E = 79N. We need $9 < C + C + E < 20$ and we want N to be as great as possible. The remaining choices for C, E and N are 0, 1, 3, 6, and 8.

If N = 8, we need C + C + E = 18 but there are no numbers remaining that satisfy that condition.

If N = 6, we would need C + C + E = 16. This can occur when C = 8 and E = 0. Therefore, the greatest value for WIN is **796**. This occurs when we add the numbers 298, 258, and 240.

FOLLOW UP: What is the least value that WIN could be under the same conditions? [703]

NOTE: *Other FOLLOW-UP problems related to some of the above can be found in our three contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.*

Division E — Mathematical Olympiads March 5, 2019 for Elementary & Middle Schools — Contest 5

SOLUTIONS AND ANSWERS

5A	382,715
5B	308,025
5C	6
5D	15
5E	32

5A **METHOD 1** *<u>Strategy</u>: Look for a pattern within the digits.*
The sum of the digits in the ones place is 15. Moving left, each column sum is 5 more than the column sum on its right. The sum of the digits in the tens place is $15 + 5 = 20$, the sum in the hundreds place is $15 + 5 + 5 = 25$, and so on. The total is $1(15) + 10(20) + 100(25) + 1000(30) + 10000(35) =$ **382,715**.

METHOD 2 *<u>Strategy</u>: Look for a pattern within the numbers.*
Notice that the second number is 11111 more than the first and the third number is 11111 more than the second and so on. The sum equals $5 \times 54321 + (11111 + 22222 + 33333 + 44444) = 271605 + 111110 = 382{,}715$.

FOLLOW UP: Find the sum of the digits in the original problem. [125]

5B **METHOD 1** *<u>Strategy</u>: Understand and apply the pattern demonstrated.*
Let n be the first digit(s) in the number being squared. For example when 45 is squared n is 4 and when 135 is squared n is 13. The squared value is $n \times (n + 1)$ followed by the number 25. Therefore when $n = 55$, the result is $55 \times 56 = 3{,}080$ followed by the number 25. The final result is **308,025**.

METHOD 2 *<u>Strategy</u>: Multiply and look for a pattern.*
Multiply 555×5 to get 2775. Then $555 \times 50 = 27750$ and $555 \times 500 = 277500$. Add these three results together to get 308,025.

FOLLOW UP: Solve for N: $125^2 = 100N + 25$ [156]

5C **METHOD 1** *<u>Strategy</u>: Use divisibility rule for 9.*
Since $99 = 9 \times 11$, the sum of the digits for the 5-digit number 2A13A must be a multiple of 9. Therefore, $2 + A + 1 + 3 + A = 2A + 6$ must be a multiple of 9. Consider the possibilities: $2A + 6 = 9$, $2A + 6 = 18$, $2A + 6 = 27$, … . The only single digit value for A occurs when $2A + 6 = 18$ and A = **6**.

METHOD 2 *<u>Strategy</u>: Use divisibility rules for 9 and 11.*
The divisibility rule for 9 is that the sum of the digits must be a multiple of 9. Therefore, $2 + A + 1 + 3 + A = 2A + 6$ must be a multiple of 9.

The divisibility rule for 11 is that alternating between subtraction and addition of the digits must be a multiple of 11. Therefore, $2 – A + 1 – 3 + A = 0$ must be a multiple of 11. Since 0 is a multiple of 11, every value of A results in 2A13A being a multiple of A.

Now apply the procedure demonstrated in Method 1.

FOLLOW UP: Suppose 4A12B is divisible by 99. How many different values for 4A12B are possible? [1]

5D **METHOD 1** *<u>Strategy</u>: Visualize placing additional cubes layer by layer.*
Beginning at the bottom, 5 additional cubes can be placed. The next layer requires 4 additional cubes. Continuing this pattern; 5 + 4 + 3 + 2 + 1 = **15** additional cubes are needed.

METHOD 2 *<u>Strategy</u>: Visualize beginning the process from an empty corner and find a pattern.*
Begin the 1st layer with a single cube to "pave" the corner. The 2nd layer requires 1 cube higher and 2 lower cubes, 1 + 2 = 3 cubes. The 3rd layer requires 1 cube higher, 2 second layer cubes, and 3 bottom cubes, 1 + 2 + 3 = 6 cubes. Continuing, the 5th layer requires 1 + 2 + 3 + 4 + 5 = 15 cubes. [Note: The set of numbers {1, 3, 6, 10, 15, …} is called the triangular numbers.]

FOLLOW UP: If she continues building, how many cubes will she need to go from a stack 19 high to 20? [210]

5E **METHOD 1** *<u>Strategy</u>: Develop a rule for ascending from a lower rung to the next higher rung.*
From the middle of any rung, there are exactly 2 ways to arrive at the next higher rung: move (left and up), or move (right and up). Thus, there are 2 ways to ascend from A to rung two, 2 ways to ascend from rung two to rung 3, and so on. The *"multiplication/counting" principle*, tells us to multiply the number of ways to perform each step. There are $2 \times 2 \times 2 \times 2 \times 2 = \mathbf{32}$ ways for the ant to ascend from A to B. Summary table of the results where 1 is the bottom rung and 6 the top rung:

Rung position	2	3	4	5	6
Number of ways to left endpoint	1	2	4	8	16
Number of ways to right endpoint	1	2	4	8	16

Arriving at point B can be accomplished in 16 + 16 = **32** ways.

METHOD 2 *<u>Strategy</u>: Recognize a pattern.*
Let L = left, U = up, and R = right, and note the number of times the ant must travel up going from rung to rung. From A to the middle of rung 2, going *up once*, the ant can go: LUR or RUL $2 = 2^1$ ways. From A to middle of rung 3, going *up twice*, the ant can go: LUUR, LURUL, RUUL or RULUR, $4 = 2^2$ ways. From A to middle of rung 4, going *up 3 times*, the ant can go: LUUUR, LUURUL, LURUUL, LURULUR, RUUUL, RUULUR, RULUUR, RULURUL, $8 = 2^3$ ways. Recognizing the pattern, there are $16 = 2^4$ ways to get to rung 5 (going *up 4 times*) and $32 = 2^5$ ways to get to rung 6 (going *up 5 times*).

METHOD 3: *<u>Strategy</u>: Simplify the problem.*
If there were only 2 rungs, there would be 2 ways to get from the middle of the bottom rung to the middle of the top rung (LUR and RUL). If there were 3 rungs, there would be 4 ways to get to the middle of the top rung (LUUR, LURUL, RULUR, and RUUL). Notice the symmetry in the 4 actions. If there were 4 rungs, there are 8 ways to climb (LUUUR, LUURUL, LURUUL, LURULUR and the 4 that swap R and L). Continuing in this fashion there are 16 ways to climb up 5 rungs and 32 ways to climb up 6 rungs. Notice that the number of rungs is always equal to the number of UP moves minus 1 and each Right and Left moves cannot be adjacent to each other.

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